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Problems 2019-3

We propose four sangaku problems taken from [1, 3]. Please send a solution with something new. There is no deadline.

Problem 1 ([1])**.** For a right triangle ABC with right angle at C, D and E are points on the side CA, F and G are points on the side AB, H is a point on the side BC and I is a point on the segment DF, where DEF is an equilateral triangle, $FGHI$ is a square and $CDIH$ is a circumscribed quadrilateral (see Figure 1). Prove or disprove $|BC| = |CE|$.

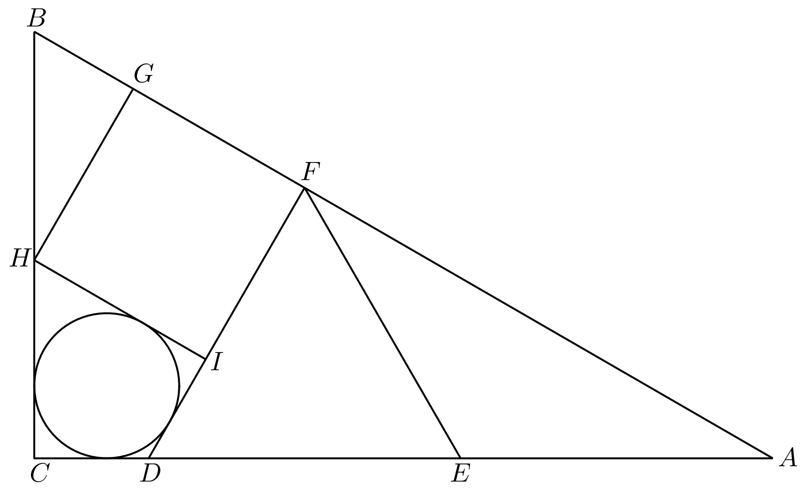

Figure 1.

Problem 2 ([1])**.** For a segment AB with midpoint M, let α, β and γ be semicircles of diameters AM, BM and AB, respectively, constructed on the same side of AB (see Figure 2). Let t be the external common tangent of α and β. ε_i $(i = 1, 2, 3, 4)$ is the circle of radius e touching t from the side opposite to M such that ε_1 and ε_2 touch externally, ε_i $(i \geq 3)$ touches ε_{i-1} at the farthest point on ε_{i-1} from ε_{i-2} and ε_1 and ε_4 touch the minor arc of γ cut by t internally. Prove or disprove that if d is the inradius of the curvilinear triangle formed by t, α and γ, then $e = 3d$ holds.

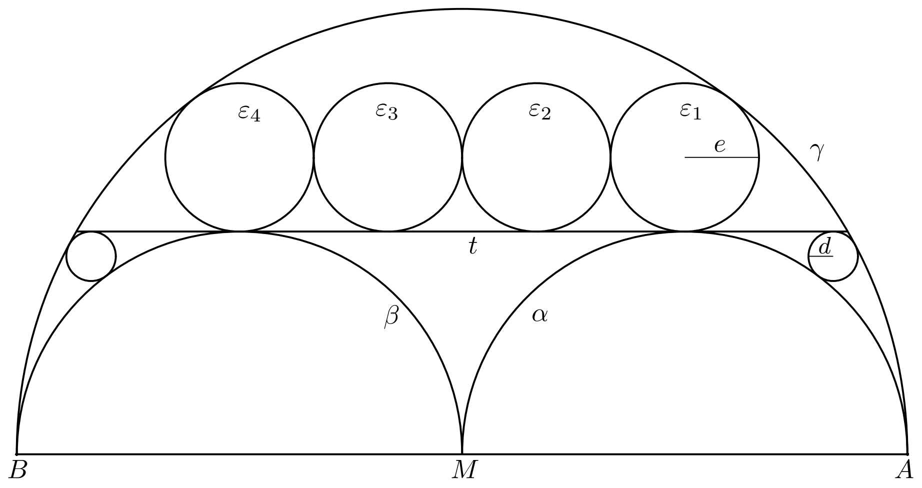

Figure 2.

We can easily see that $3e$ equals the radius of α. Therefore ε_1 or ε_4 touches α. This implies that Problem 2 is essentially the same to Satoh's problem considered in [2].

Problem 3 ([3])**.** Let α, β and γ be mutually touching three circles with collinear centers, where α and β touch γ internally and have radii a and b, respectively (see Figure 3). Let t be one of the external common tangents of α and β and let γ_t be the minor arc of γ cut by t. ε_2 is a circle touching γ_t internally at the midpoint from the inside of γ, ε_1 and ε_3 are the congruent circles touching t, ε_2 externally γ_t internally. Prove or disprove that if the circles ε_1 and ε_2 are also congruent and have radius r, then

$$r = \frac{ab(a+b)}{a^2+3ab+b^2}$$

holds.

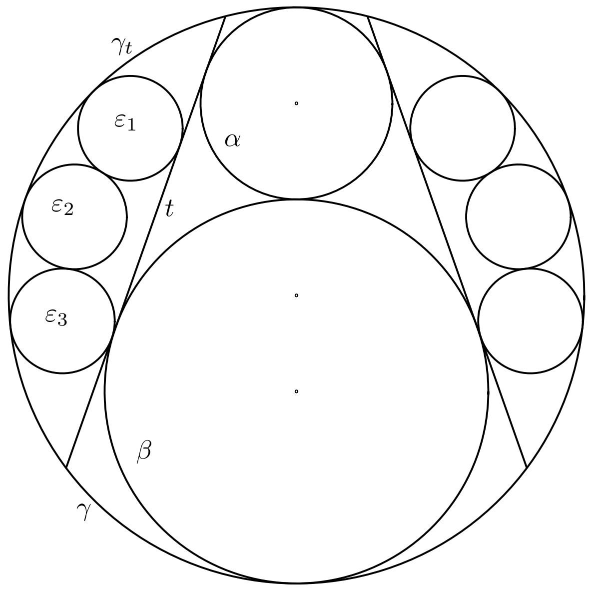

Figure 3.

Problem 4 ([3])**.** For a triangle ABC with circumcircle δ, let ε_a be the circle touching the side BC at the midpoint and the minor arc BC of δ internally (see Figure 4). Let r_a be the inradius of the two curvilinear triangles made by δ, ε_a and the side BC, and we define r_b and r_c similarly. Prove or disprove that $r_c = r_a + r_b$ holds if $\angle C$ is a right angle.

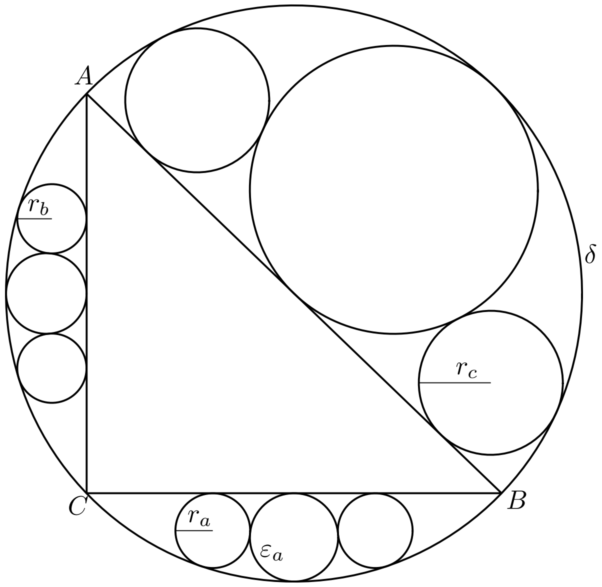

Figure 4.

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[2] H. Okumura, A note on the arbelos in Wasan geometry, Satoh's problem, Sangaku J. Math., **3** (2019) 15–16.
[3] Saitama prefectural library (埼玉県立図書館) ed., The sangaku in Saitama (埼玉の算額), 1969 Saitama Prefectural Library (埼玉県立図書館).