Sangaku Journal of Mathematics (SJM) ©SJM
Volume 1 (2017), pp.1-6
web: http://www.sangaku-journal.eu/

Japanese mathematics[2]

Hiroshi Okumura
Department of Information Engineering, Maebashi Institute of Technology, 460-1 Kamisadori Maebashi Gunma 371-0816, Japan.
e-mail: okumura@maebashi-it.ac.jp

1. Introduction

Japanese mathematics developed during the Edo period (1603-1867) is called *wasan.* It is based on Chinese mathematics books brought back to Japan in late 16^{th} century when Hideyoshi, the ruler of Japan at that time, invaded the Korean peninsular. Takakazu (or Kowa) Seki (1642?-1708) improved the Chinese way of algebraic calculation of one determinate and made it possible to solve equations with a number of unknown. After that Japanese mathematics developed very rapidly in it own original way.

There are two customs which accelerated the development of wasan, One is *idai* which is a challenging problem at the end of wasan books. When wasan mathematicians published a book, they proposed unsolved problems at the end of the book. Then others, who succeeded in solving the problems, published their solutions with other challenging problems at the end of their books. Seki's attempt to improve algebraic calculation was also made when he tried to solve such a challenging problem. Another custom is a *sangaku*, a wooden tablet of mathematics. When people found interesting properties or solved hard problems, they wrote them as problems on framed wooden boards and dedicated them to a shrine or a temple. Then they would be hung under the roof. Most such problems were geometric and the figures were beautifully drawn in color. It was also a means to publish discoveries or to propose new problems.

Japan's feudal government closed the country during the Edo period. But at the beginning of the Meiji era (1868-1912) the new government opened the country and adopted Western mathematics in the new schooling system. There after wasan followed a course of decline. In this article we will briefly demonstrate

[2]This article is reprinted from *Symmetry: Culture and Science* **12**(1-2) 79–86 (2001) with permission from the publisher.

the mathematics of wasan and will also show the position of wasan today. For extensive references see (Mikami 1913) and (Smith and Mikami 1914).

2. Examples

Roughly speaking wasan covers a part of analysis, number theory, combinatorics and geometry. The area of circles, length of circular arc, volume of intersecting solids, solutions to indefinite equation and magic squares are popular wasan topics. Also wasan mathematicians studied astronomy, surveying and the art of divination in many cases. Though they studied certain aspects of some things deeply, they did not establish theoretical system. For example ellipses were studied often, but hyperbolas and parabolas were not. Also rhombuses were considered often but parallelograms barring a couple of exceptions, were not. Since almost all wasan books were written as problem books following the tradition of Chinese mathematics books, it was not necessary to treat particular topics. On the other hand there are also wasan books which treated particular subjects (Okumura 1999, this journal). Most geometric problems attempt to discover certain dimensions of geometric figures such as the radius of a circle, a major (or minor) axis of an ellipse, the side of a square, etc.

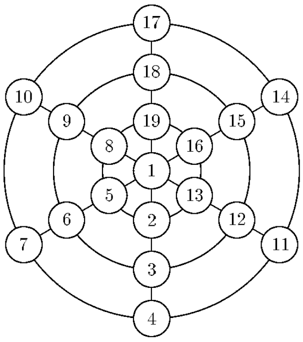

Figure 1.

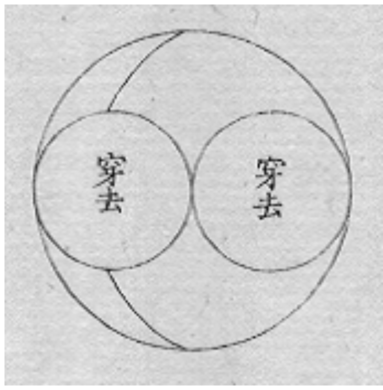

Figure 2.

Let's examine some examples. Figure 1 shows a magic circle similar to a magic square (Seki). It consists of n concentric circles and n lines through the center, where natural numbers 1, 2, $\cdots$, $2n^2+1$ are located at the intersections of the circles and the lines so that the sum of the numbers located on each of the circles and the center, is equal to the sum of the numbers located on each of the lines. The idea comes from antecedent Chinese books. Seki shows a general construction of magic circles.

In the early stages of wasan, the area of a circle and length of a circular arc were popular problems. But in the later stage, the volume of intersecting solids and surface areas were often considered. Figure 2 shows the Viviani's problem in wasan: A sphere of radius r is bored by two cylinders with the radii $r/2$ as in the figure. Find the volume and the surface area of the remaining part (Uchida 1844).

Sangaku problem may be very useful and enjoyable in classroom. But it also seems that sangaku problems are still attractive mathematically today, since they are so unhackneyed and challenging. Let us see two such problems.

Problem 1. Five circles of three different sizes touch as in Figure 3. Given the radius of the largest circle, find the radius of the medium circle (Saitama prefectural library 1969). The answer is that the medium circle is half the size of the largest circle.

Problem 2. In a square $PQRS$, there are two circles touching SP and the incircle of the square, where one of which touches PQ and the other touches RS. Let A be the point of tangency of QR and the incircle and let the tangents of the two small circles through A intersect the segment SP at B and C as in Figure 4. Given the inradius of the square, find the inradius of the triangle ABC (Hirayama and Matsuoka 1969). The answer is that the medium circle is also half the size of the largest circle.

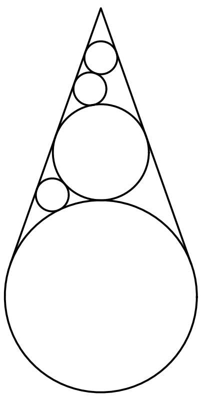

Figure 3.

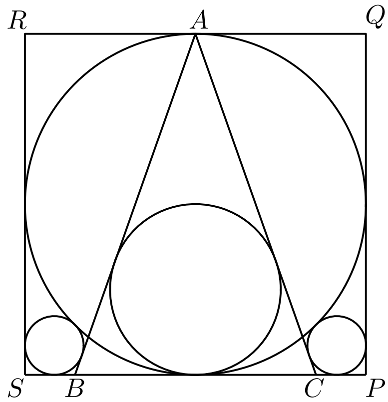

Figure 4.

3. Sangaku problems today

Most sangaku problems are proposed as problems and their final solutions. There are no explanations about how we can derive the final solution. Hence it is of some worth to solve the problem and to find the process in order to derive the final answer. On the other hand we can sometimes find interesting properties or interesting configurations from sangaku figures. An example can be seen in Okumura 1997. Here we will show some such examples using the two sangaku problems mentioned in the previous section.

In Problem 1, there are three small congruent circles, two of which lie in the curvilinear triangles made by two external tangents and the medium circle. The other lies in one of the curvilinear triangles made by the two larger circles and one of the external tangents. The problem says that the ratio of the size of the two larger circles is $2:1$. Considering the problem, we found the following surprising fact (Okumura and Sodeyama 1998): If there are two externally touching circles with different radii with external common tangents and if there are $4n$ congruent small circles, n of which lie in the curvilinear triangle made by two external tangents and the medium circle, and $3n$ of which lie in one of the curvilinear triangles made by the two larger circles and one of the external tangents as in Figure 5, then the ratio of the size of the two larger circles is $4:1$ for any natural number n. Figure 5 shows the cases $n = 1, 2$.

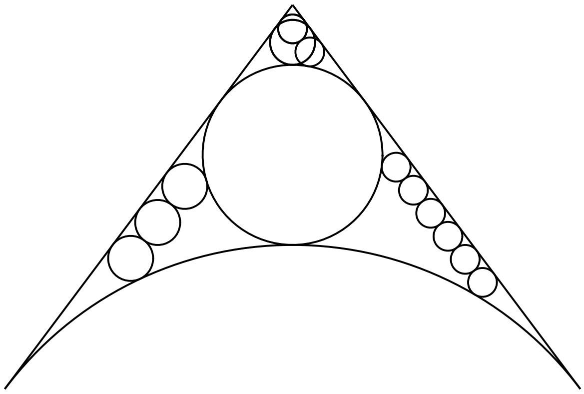

Figure 5.

In Problem 2, $PQRS$ is a square. But if $PQRS$ is a rhombus (see Figure 6), we can still reach the same conclusion, that is, the incircle of the triangle ABC is half the size of the incircle of the rhombus (Okumura and Nakajima 1998).

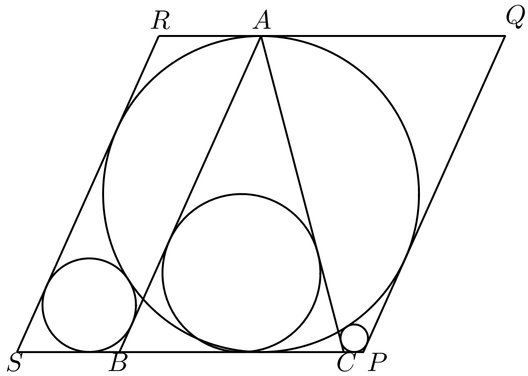

Figure 6.

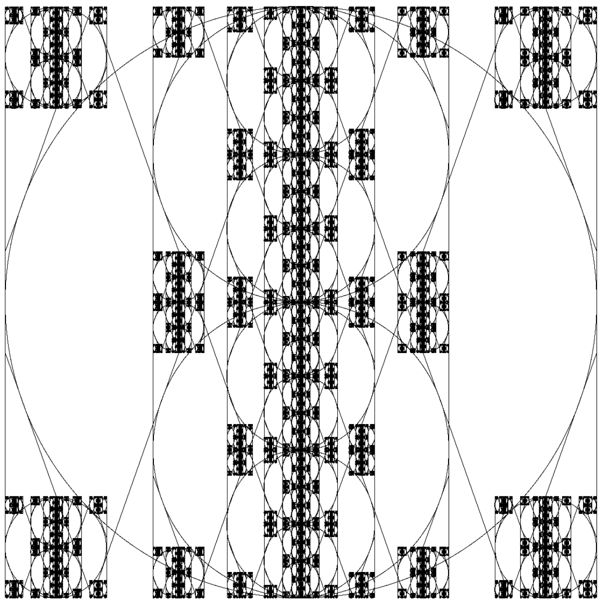

Figure 7.

Another interesting result of this problem is a fractal image (see Figure 7), which is made bases on the fact that the ratio of the size of the two inscribed circles in the problem is 2 : 1. This provides an example of recursive computer programming. An example using sangaku figures in computer animation can be seen in (Okumura, proc. 1999).

The author totally agree with the following comments (Rigby 1999): There may be a need for a book whose main aims are: (1) to reduce the need for repetitious proofs by grouping related results together, (2) to simplify the presentation of results, (3) to simplify proofs when possible, (4) to generalize some of the results, which can sometimes lead to greater insight and to simpler proofs.

4. Wasan today

Most wasan books are written in Chinese, which makes them rather hard to read for ordinary Japanese today. In addition to this, wasan books have scarcity value, making them very expensive and difficult to find in bookstore. Even if we could find a library, where some wasan source materials are collected, they are still much more difficult to read than other mathematics books. We have to settle certain bureaucratic matters before getting desired books. Therefore most Japanese have not seen wasan books. They know the word wasan, but do not know what problems there are, even if they are mathematics teachers. It is extremely regrettable that wasan is not so popular in Japan and is hardly used in classroom lessons.

There are many wasan problems; some are interesting and some are not. Recently Fukagawa published wasan problem books both in Japanese and in English, with coauthors (Fukagawa and Pedoe 1989; Fukagawa and Sokolowsky 1994). We can see interesting problems in these books. But the difficulty to access wasan source material is still unsettled. Only those who can see wasan source materials have been able to study wasan for a long time. But a recent publication provides wasan source material in a set of six CD-ROMs (Okumura 2001). Though the book of explanation is written in Japanese, the main data consists of pictures in JPEG format so they can be seen on most computers with any language. This set covers more than three hundreds wasan source material. Also we can see several books of wasan and Chinese mathematics at the web site of Kyoto University. There also are several web sites in English, where we can see some sangaku problems. We can access these site from most portal sites by using the key word "sangaku". We can see wasan source material more easily than before, but there are still many aspects of wasan which need to be studied both historically and mathematically.

References

Fukagawa, H. and Pedoe, D. (1989) *Japanese Temple Geometry Problems*, Charles Babbage Research Center, Winnipeg, Canada.

Fukagawa, H. and Sokolowsky, D. (1994) *Japanese mathematics - how many problems can you solve?*, Morikita Shuppan, (in Japanese).

Hirayama, A. and Matsuoka, M. (1966) *The sangaku in Yamagata*, (private publication in Japanese).

Mikam, Y. (1913) *The development of Mathematics in China and Japan*, Chelsea, N. Y.

Okumura, H. (1997) Circle patterns arising from results in Japanese geometry, *Symmetry: Culture and Science*, **8**(1) 4–23.

Okumura. H. and Sodeyama, C. (1998) A surprising property of successively touching circles, *Mathematics Plus*, **6**(3) 17–18 (in Bulgarian).

Okumura, H. and Nakajima, E. (1998) Incircles of rhombuses, *Mathematics and Informatics Quarterly*, **8**(3) 143–148.

Okumura, H. (1999) Mathematical study of wasan geometry, *Proc. of the 4th international symposium on the history of mathematics and mathematics education using Chinese characters* 129–138.

Okumura, H. (1999) Geometries in the East and the West in the 19^{th} century, *Symmetry: Culture and Science*, **9**,(1-2) 189–197.

Okumura, H., ed. (2001) *Wasansho Shusei* (wasan collection), Iwanami Shoten, Tokyo (in Japanese).

Rigby, J. (1999) Some Western thoughts on traditional Japanese mathematics, *Proc. of the 4th international symposium on the history of mathematics and mathematics education using Chinese characters*, 163–175.

Saitama prefectural Library (1969) *The sangaku in Saitama*, Saitama prefectural Library, Saitama Japan, (in Japanese).

Seki, *Hojin no ho ensan no ho* (apograph).

Smith, D. E. and Mikamim Y. (1914) *A history of Japanese mathematics.* Open Court, Chicag.

Uchida (1844) *Sampo Kyuseki Tsuko.*