Problems 2019-4

Please send a solution with something new idea or a generalization.

Problem 1 ([1]). For a square $P_1P_2P_3P_4$, let Q_i ($i = 1, 2, 3, 4$) be a point on the side P_iP_{i+1} such that $Q_1Q_3 \perp Q_2Q_4$, where the subscripts are taken modulo 4. Let r_i ($i = 1, 2, 3, 4$) be the radius of the circle lying inside of the square and touching $P_iP_{i\pm1}$ and Q_iQ_{i+2}. Prove or disprove $r_1 + r_3 = r_2 + r_4$ (see Figure 1).
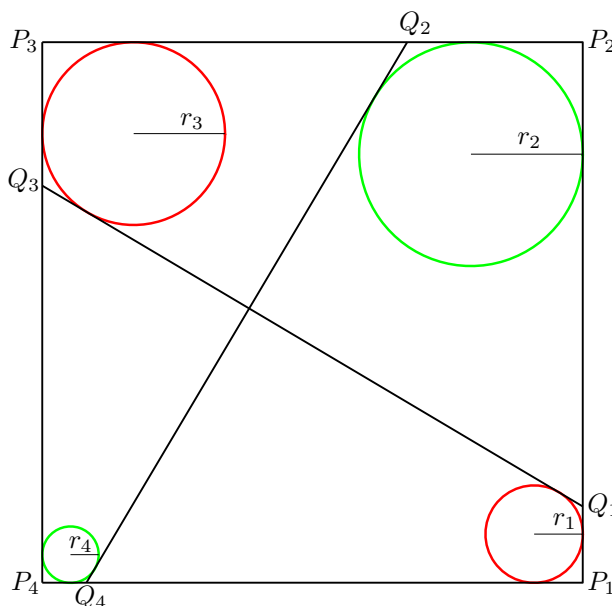

Figure 1.

References

[1] Sampō Zatsurui (算法雑類), Tohoku University Digital Collection.