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## Problem 2020-2

ABDILKADIR ALTINTAS Emirdağ Anadolu Lisesi, Emirdağ AFYON, Turkey e-mail: kadiraltintas1977@gmail.com

Editor's comments. Generalizations and farther comments for this problem are not required. There is no deadline of submission.



**Problem 1.** For a point O on the segment AB, let  $\alpha$ ,  $\beta$  and  $\gamma$  be the semicircles of diameters AO, BO and AB, respectively, where the three semicircles form an arbelos and |AO| = 2a and |BO| = 2b (see Figure 1). Assume that the incircle of the arbelos touches  $\beta$  and  $\gamma$  at points C and D, respectively, and the line CD meets AB in a point E. Let  $\angle BCE = \theta$  and  $\angle BDE = \phi$ . Prove or disprove that the following relations hold.

(1)  $\cot \theta = \frac{b}{a}$ . (2)  $\cot \phi = 1 + \frac{b}{a}$ .

**Remark.** Assume that C' is the point of tangency of the incircle and the semicircle  $\alpha$  and the line C'D meets AB in a point E'. Let  $\angle AC'E' = \theta'$ . Notice that if (1) and (2) are true, the followings hold.

(i)  $\cot \phi - \cot \theta = 1$ .

(ii)  $\theta + \theta' = 90^{\circ}$ .

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