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Problem 2020-2

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Editor's comments. Generalizations and farther comments for this problem are not required. There is no deadline of submission.

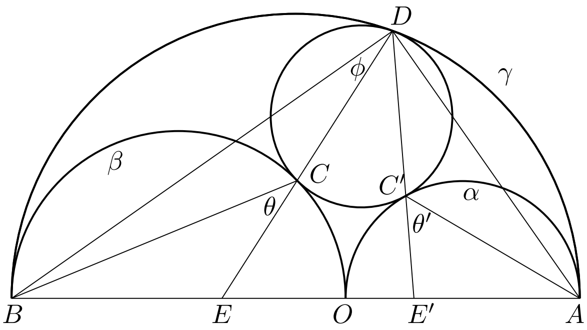

Figure 1.

Problem 1. For a point O on the segment AB, let α, β and γ be the semicircles of diameters AO, BO and AB, respectively, where the three semicircles form an arbelos and $|AO| = 2a$ and $|BO| = 2b$ (see Figure 1). Assume that the incircle of the arbelos touches β and γ at points C and D, respectively, and the line CD meets AB in a point E. Let $\angle BCE = \theta$ and $\angle BDE = \phi$. Prove or disprove that the following relations hold.

(1) $\cot\theta = \dfrac{b}{a}$.

(2) $\cot\phi = 1 + \dfrac{b}{a}$.

Remark. Assume that C' is the point of tangency of the incircle and the semicircle α and the line $C'D$ meets AB in a point E'. Let $\angle AC'E' = \theta'$. Notice that if (1) and (2) are true, the followings hold.

(i) $\cot\phi - \cot\theta = 1$.

(ii) $\theta + \theta' = 90°$.